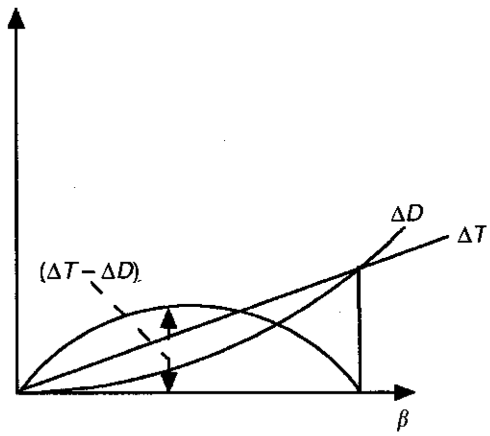


FIGURE 5.3 Thrust and drag variation of a turbofan engine with BPR (β).

which corresponds to the maximum difference between the increased thrust and the increased drag ($\Delta T - \Delta D$), the net thrust gain.

If, on the other hand, takeoff thrust is a very important consideration, then, it might be worthwhile to increase the BPR beyond this value, since, at takeoff, the drag would be small.

Moreover, both the jet velocities of the cold air from the fan nozzle and the hot gases from the turbine nozzle are less than the jet velocity of turbojet engines. This represents an additional advantage of turbofan engine over simple turbojet engines as the jet noise is correlated to the jet velocity. The intensity of jet noise has been shown to be proportional to the eighth power of the velocity of the jet relative to the ambient air, thus small reductions in velocity means significant reductions in noise. It is well known that, the noise of the jet engine, especially those noise components with frequencies disagreeable to humans, appear to come from the compressor.

Another possible advantage of turbofan engines over the turbojet ones is associated with the lower temperature of the hot exhausts before the nozzle. Thus, the exhaust is then much less susceptible to infrared detection.

5.3 FORWARD FAN UNMIXED TWO-SPOOL ENGINES

5.3.1 The Fan and Low-Pressure Compressor on One Shaft

The famous engine of this configuration is the General Electric CF6 engine series. This engine is composed of a low-pressure spool with a single-stage fan and a three-stage low-pressure compressor (LPC) (sometimes denoted as booster by aero engines manufacturers). Both are driven by a five-stage low-pressure turbine (LPT). The high-pressure spool is composed of a 14-stage high-pressure compressor (HPC) driven by a two-stage high-pressure turbine (HPT).

Another famous example is Pratt and Whitney PW4000 series.

A typical turbofan of this configuration together with the corresponding T–S diagram is shown in Figures 5.4 and 5.5.

The different modules are analyzed here.

1. *Intake:* The same governing equations for the intake in the single-spool configuration, that is, Equations 5.1 and 5.2 are applied here.

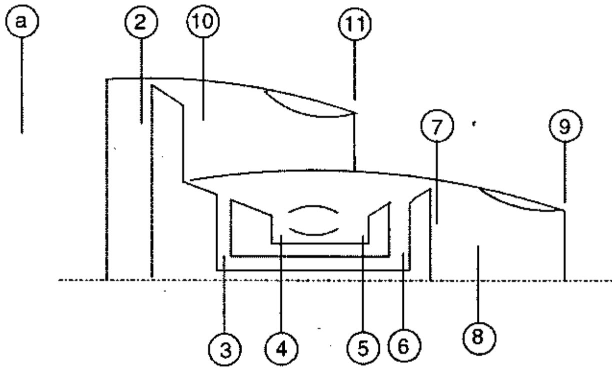


FIGURE 5.4 Layout of a two-spool turbofan engine (fan and compressor driven by LPT).

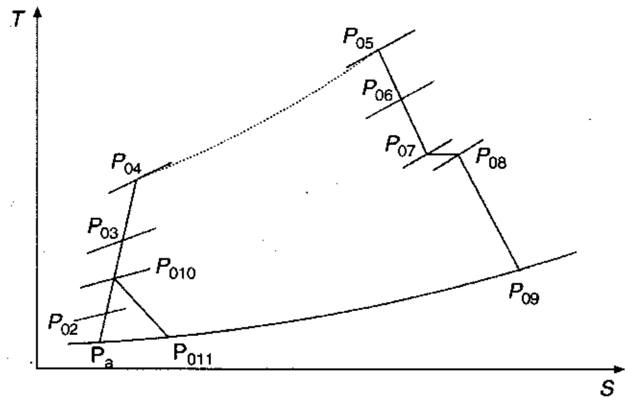


FIGURE 5.5 Temperature–entropy diagram of a two-spool turbofan engine.

2. *Fan:* The following equations are applied:

$$P_{010} = (P_{02})(\pi_f) \tag{5.16}$$

$$T_{010} = T_{02}\left[1 + \frac{1}{\eta_f}\left(\pi_f^{(\gamma-1)/\gamma} - 1\right)\right] \tag{5.17}$$

3. *Low-pressure compressor:* The pressure ratio achieved in this compressor is less than 2.

$$P_{03} = (P_{010})(\pi_{LPC}) \tag{5.18}$$

$$T_{03} = T_{010}\left[1 + \frac{1}{\eta_{LPC}}\left(\pi_{LPC}^{(\gamma-1)/\gamma} - 1\right)\right] \tag{5.19}$$

4. *High-pressure compressor:*

$$P_{04} = (P_{03})(\pi_{HPC}) \tag{5.20}$$

$$T_{04} = T_{03}\left[1 + \frac{1}{\eta_{HPC}}\left(\pi_{HPC}^{(\gamma-1)/\gamma} - 1\right)\right] \tag{5.21}$$

In Equations 5.17 through 5.21, $\gamma \equiv \gamma_c$.

5. *Combustion chamber:* The pressure at the outlet of the combustion chamber is obtained from the pressure drop in the combustion chamber

$$P_{05} = P_{04} - \Delta P_{cc} \tag{5.22a}$$

or

$$P_{05} = P_{04}(1 - \Delta P_{cc}\%) \tag{5.22b}$$

The temperature at the outlet of the combustion chamber is also the maximum temperature in the engine and known in advance. Thus, the fuel-to-air ratio is calculated from the relation

$$f = \frac{(Cp_h/Cp_c)\ (T_{05}/T_{04}) - 1}{\eta_b\ (Q_R/Cp_c T_{04}) - (Cp_h/Cp_c)\ (T_{05}/T_{04})} \tag{5.23}$$

6. *High-pressure turbine:* To calculate the temperature and pressure at the outlet of the HPT, an energy balance between the HPC and HPT is expressed by the relation

$$\dot{m}_a\, Cp_c\ (T_{04} - T_{03}) = \dot{m}_a\ (1+f)\, Cp_h\ (T_{05} - T_{06}) \tag{5.24}$$

From the above relation, the temperature at the outlet of the turbine T_{06} is defined. Moreover, from the known isentropic efficiency of HPT (η_{HPT}) then the outlet pressure from the HPT (P_{06}) may be obtained as explained above in the single-spool turbofan (Equations 5.11a and 5.11b).

7. *Low-pressure turbine:* An energy balance between the fan and LPC from one side and the LPT on the other side is expressed by the relation

$$\beta \dot{m}_a Cp_c\,(T_{010} - T_{02}) + \dot{m}_a\, Cp_c\ (T_{03} - T_{02}) = \dot{m}_a\ (1+f)\, Cp_h\ (T_{06} - T_{07}) \tag{5.25a}$$

or

$$(1+\beta)\ \dot{m}_a Cp_c\,(T_{010} - T_{02}) + \dot{m}_a\, Cp_c\ (T_{03} - T_{010}) = \dot{m}_a\ (1+f)\, Cp_h\ (T_{06} - T_{07}) \tag{5.25b}$$

$$\therefore T_{07} = T_{06} - \left(\frac{1+\beta}{1+f}\right)\left(\frac{Cp_c}{Cp_h}\right)(T_{010} - T_{02}) - \left(\frac{Cp_c}{Cp_h}\right)\left(\frac{T_{03} - T_{010}}{1+f}\right)$$

The pressure at the outlet is obtained from the relation

$$P_{07} = P_{06}\left(1 - \frac{T_{06} - T_{07}}{\eta_{t2} \times T_{06}}\right)^{\gamma_h/(\gamma_h - 1)}$$

Now, if there is an *air bleed* from the HPC at a station where the pressure is P_{03b}, then the energy balance with the HPT gives

$$\dot{m}_a Cp_c (T_{03b} - T_{03}) + \dot{m}_a (1 - b) (T_{04} - T_{03b}) = \dot{m}_a (1 + f - b) Cp_h (T_{05} - T_{06}) \tag{5.26}$$

where $b = \dot{m}_b/\dot{m}_a$ is the air bleed ratio defined as the ratio between the air bled from the HPC and the core air flow rate.
Moreover, such a bleed has its impact on the energy balance of the low-pressure spool as the air passing through the LPT is now reduced.

$$(1 + \beta) \dot{m}_a Cp_c (T_{010} - T_{02}) + \dot{m}_a Cp_c (T_{03} - T_{010}) = \dot{m}_a (1 + f - b) Cp_h (T_{06} - T_{07}) \tag{5.27}$$

The flow in the jet pipe is frequently associated with a pressure drop mainly due to skin friction.
Thus the pressure upstream of the turbine nozzle is slightly less than the outlet pressure from the turbine. The temperature however, is the same. Thus

$$P_{08} = P_{07} \left(1 - \Delta P_{\text{jet pipe}}\right)$$
$$T_{08} = T_{07}$$

Concerning the other modules or components, a similar procedure is followed. The exhaust velocities of both the cold air from the fan nozzle and the hot gases from the turbine nozzles are obtained after checks for choking. The thrust force, specific thrust, and the TSFC are calculated.

Example 1 A two-spool turbofan engine is to be examined here. The low-pressure spool is composed of a turbine driving the fan and the LPC. The high-pressure spool is composed of a HPC and a HPT. Air is bled from an intermediate state in the HPC.

The total pressure (in psia) and total temperature (in °C) during a ground test ($M = 0.0$) are recorded and shown in the following table:

Station	Fan Inlet	Fan Outlet	LPC Outlet	Air Bleed Location	HPC Outlet	CC Outlet	HPT Outlet	LPT Outlet
P_0 (psia)	14.7	23.2	35.6	110	347	332	86.4	21.4
T_0 (°C)	15	62.7	109.3	275	501	1286	906	512

The fan and turbine nozzles have isentropic efficiency of 0.9.
It is required to calculate the following:

(a) The fan isentropic efficiency (stations 2–F2.5)
(b) The high-pressure ratio isentropic efficiency (stations 3–4)
(c) The fuel-to-air ratio (f)

5.5 FORWARD FAN MIXED-FLOW ENGINE

Mixed turbofan engines are always found in either single- or two-spool engines. It was used in the past for military applications only. Nowadays it is used in both civil and military aircraft. An example for a two-spool engine is the CFM56 series. The cold compressed air leaving the fan will not be directly exhausted as previously described but it flows in a long duct surrounding the engine core and then mixes with the hot gases leaving the LPT. Thus the cold air is heated while the hot gases are cooled. Only one mixed exhaust is found.

A layout of a single-spool mixed turbofan and its T–S diagram is shown in Figures 5.16 and 5.17.

5.5.1 Mixed-Flow Two-Spool Engine

Most of the mixed turbofan engines now are two-spool ones. If mixed turbofan engines are analyzed versus unmixed turbofan engines, the following points are found [6]:

1. The optimum fan pressure ratio for a mixed-flow engine is generally lower than that for a separate flow for a given BPR.
2. At a given fan pressure ratio, the mixed-flow engine has a lower BPR and therefore a higher specific thrust.
3. The amount of power that the LPT supplies to drive the fan will be smaller.
4. Possibly one LPT stage less is sufficient.
5. Other features to be considered are
   - Thrust gain due to mixing
   - Noise
   - Weight
   - Reverse thrust.

Hereafter, a detailed analysis of this category will be given. Figures 5.18 and 5.19 present the engine layout and its T–S diagram.

The requirements for the mixing process are equal static pressures and also equal velocities. Thus, from the layout designation these two conditions are specified as $P'_3 = P_7$ and $V'_3 = V_7$, which means that if no pressure losses in the bypass duct connecting the cold and hot streams and no pressure loss in the mixing process occur, then

$$P_{03} = P'_{03} = P_{07} = P_{08} \tag{5.34}$$

If losses occur in the fan bypass duct, then

$$P'_{03} = P_{03} - \Delta P_{\text{fan/duct}}$$
$$P_{07} = P_{08} = P_{03} - \Delta P_{\text{fan/duct}}$$

In the above equation, no pressure drop is considered during the mixing process.

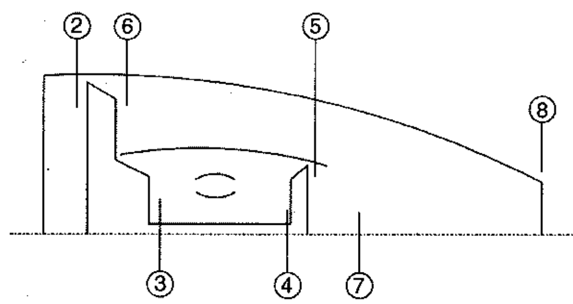


FIGURE 5.16 Single-spool mixed-flow turbofan.

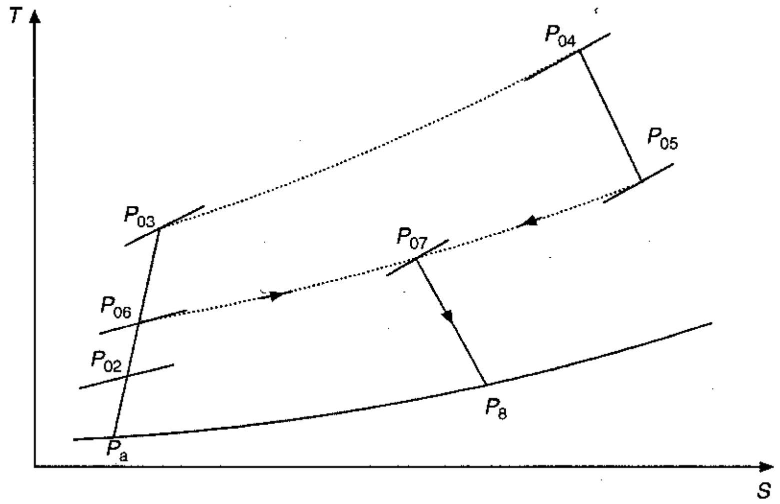


FIGURE 5.17 T–S diagram of single-spool mixed-flow turbofan.

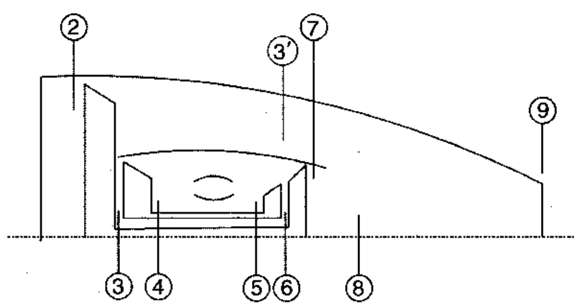


FIGURE 5.18 Layout of a mixed two-spool turbofan.

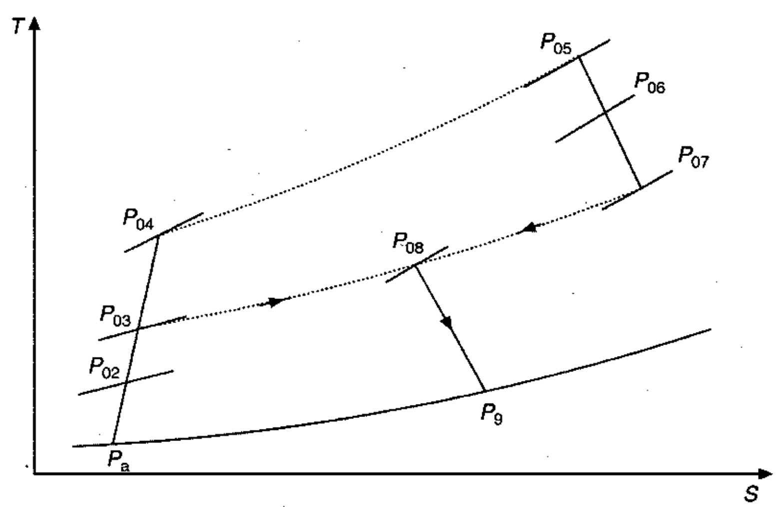


FIGURE 5.19 T–S diagram for two-spool mixed turbofan.

In some engines like the CFM56 series the mixing process takes place in a mixer preceding the nozzle. It results in a quieter engine than if the mixer was not present. Figure 5.20 illustrates a typical mixed two-spool engine.

1. *Energy balance for the low-pressure spool*: Considering a mechanical efficiency for the low-pressure spool of (η_{m1}), then

$$W_{fan} = \eta_{m1} W_{LPT}$$

or

$$\dot{m}_a (1+\beta) Cp_c (T_{03} - T_{02}) = \eta_{m1} \dot{m}_a (1+f) Cp_h (T_{06} - T_{07}) \quad (5.35)$$

2. *Energy balance for the high-pressure spool:*
Also a second mechanical efficiency for the high-pressure spool is (η_{m2}) assumed.

$$W_{HPC} = \eta_{m2} (1+f) \dot{m}_a Cp_h (T_{05} - T_{06})$$

Thus

$$\dot{m}_a Cp_c (T_{04} - T_{03}) = \eta_{m2} \dot{m}_a (1+f) Cp_h (T_{05} - T_{06}) \quad (5.36)$$

3. *Mixing process*: The hot gases leaving the LPT and the cold air leaving the fan bypass duct are mixed and give new properties at state (8). Thus such a process is governed by the first law of thermodynamics as follows:

$$H_{03} + H_{07} = H_{08}$$
$$\dot{m}_c Cp_c T_{03} + (1+f) \dot{m}_h Cp_h T_{07} = [\dot{m}_c + (1+f) \dot{m}_h] Cp_h T_{08}$$

which is reduced to

$$\beta Cp_c T_{03} + (1+f) Cp_h T_{07} = (1+\beta+f) Cp_h T_{08} \quad (5.37)$$

Now for a better evaluation of the gas properties after mixing we can use mass-weighted average properties of the gases at state (8) as follows:

$$Cp_8 = \frac{Cp_7 + \beta Cp_3}{1+\beta}$$

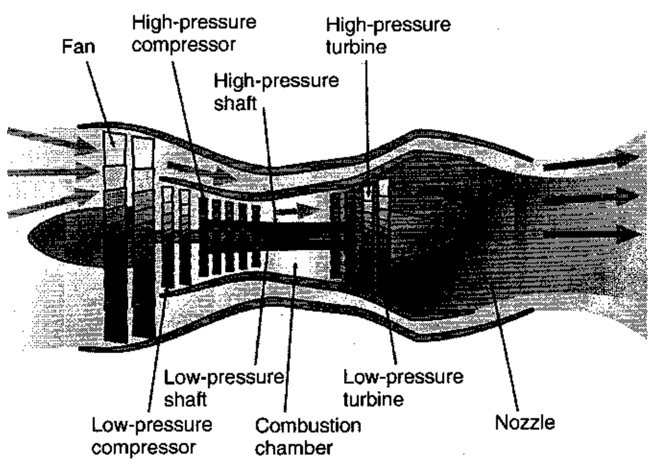


FIGURE 5.20 Two-spool mixed turbofan engine with a mixer.

$$R_8 = \frac{R_7 + \beta R_3}{1+\beta}$$

$$\gamma_8 = \frac{Cp_8}{Cp_8 - R_8}$$

Consider the real case of mixing where normally losses are encountered, where pressure drop is associated with the mixing process. Such a pressure loss is either given as the value ΔP_{mixing} or as a ratio r_{m} in the mixing process

$$P_{08} = P_{07} - \Delta P_{\text{mixing}} \quad \text{or} \quad P_{08} = r_{\text{m}} P_{07} \quad \text{where} r_{\text{m}} < 1 \approx 0.98$$

The thrust force for unchoked nozzle is then given by the relation

$$T = \dot{m}_{\text{a}} \left[(1 + f + \beta)\, U_{\text{e}} - (1+\beta)\, U \right] \tag{5.38}$$

Example 4 Figure 5.21 illustrates the mixed high BPR turbofan engine CFM56-5C. It has the following data:

BPR	6.6
Typical TSFC	0.04 kg/(N · h)
Cruise thrust	29,360 N
Total mass flow rate at cruise	100 kg/s
During takeoff operation	
Takeoff thrust	140,000 N
Total air mass flow rate at takeoff	474.0 kg/s
Overall pressure ratio	37.5
Total temperature at inlet to the LPT	1251 K
Exhaust gas temperature	349 K
Ambient temperature	288.0 K
Ambient pressure	101 kPa

Assuming that all the processes are ideal and there are no losses in both the combustion chamber and mixing process

A. Prove that the thrust force is given by the relation

$$T = \frac{\dot{m}_{\text{a}} (1+\beta)(u_{\text{e}} - u)}{1 - \text{TSFC} \times u_{\text{e}}}$$

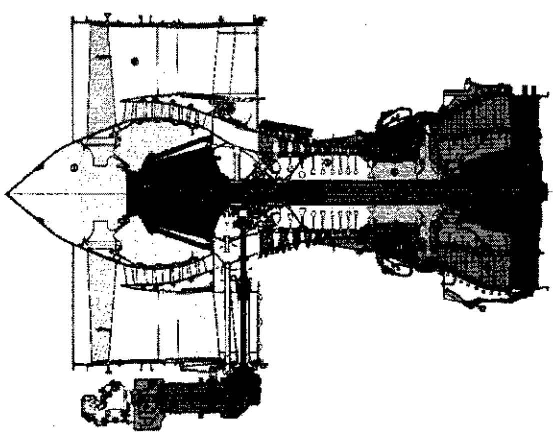


FIGURE 5.21 CFM56-5C mixed high BPR turbofan engine. (Courtesy GE Aviation.)

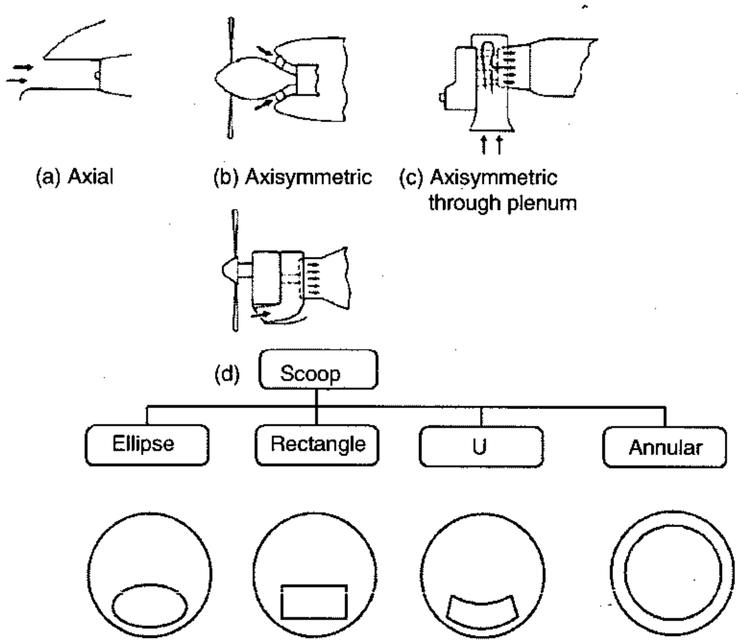


FIGURE 6.8 Types of inlets.

6.3 THERMODYNAMIC ANALYSIS OF TURBOPROP ENGINES

The different modules for a turboprop engines are the intake or inlet, one or two compressors, a combustion chamber and one or more (up to three) turbines, and the exhaust nozzle. For a single- or double-spool engine only one compressor is found. Two compressors are found in a triple-spool engine. One turbine exists for each spool.

6.3.1 Single-Spool Turboprop

A simplified layout of a single-spool turboprop engines with its different states and the corresponding temperature–entropy diagram is shown in Figures 6.9 and 6.10. The same procedure followed in previous chapters will be followed.

The flight speed is expressed as $U = M_a\sqrt{\gamma R T_a}$.

The thermodynamic properties at different locations within the engine are obtained as follows:

$$\gamma_c = \frac{Cp_c}{(Cp_c - R)}, \quad \gamma_{cc} = \frac{Cp_{cc}}{(Cp_{cc} - R)}, \quad \gamma_t = \frac{Cp_t}{(Cp_t - R)}, \quad \gamma_n = \frac{Cp_n}{Cp_n - R}$$

The different modules of the engine are treated as previously described in Chapters 3, 4, and 5.

1. *Intake:* The intake has an isentropic efficiency (η_d); the ambient temperature and pressure are T_a and P_a respectively and the flight Mach number is M_a. The temperature and pressure at the intake outlet, T_{02} and P_{02}, are given by the following relations:

$$P_{02} = P_a\left(1 + \eta_d\frac{\gamma_c - 1}{2}M_a^2\right)^{\gamma_c/(\gamma_c-1)} \tag{6.1}$$

$$T_{02} = T_a\left(1 + \frac{\gamma_c - 1}{2}M_a^2\right) \tag{6.2}$$

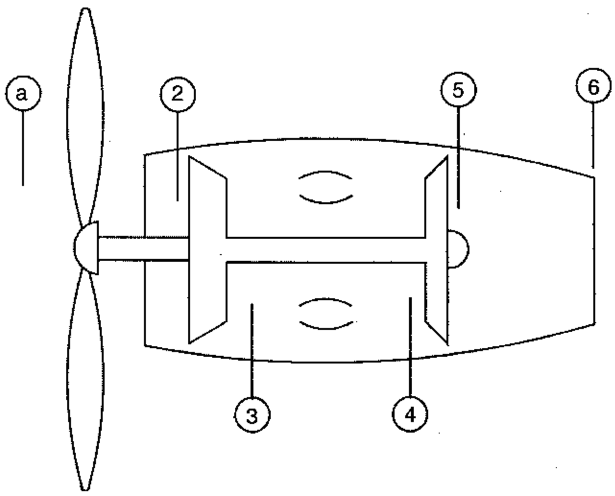


FIGURE 6.9 Layout of a single spool (direct drive turboprop engines).

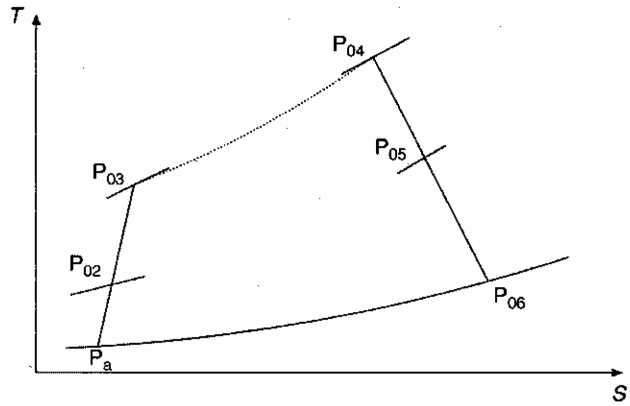


FIGURE 6.10 Temperature–entropy diagram of single spool turboprop.

2. *Compressor:* For a known compressor pressure ratio (π_c) its isentropic efficiency is (η_c); thus the pressure and temperature at the outlet of the compressor as well as the specific power of the compressor are given by the following relations:

$$P_{03} = (P_{02})(\pi_c) \tag{6.3}$$

$$T_{03} = T_{02}\left[1 + \frac{\pi_c^{\frac{\gamma_c - 1}{\gamma_c}} - 1}{\eta_c}\right] \tag{6.4}$$

$$\Delta h_c = Cp_c(T_{03} - T_{02})$$

3. *Combustion chamber:* The combustion process takes place in the combustor with an efficiency of (η_b), while the products of combustion experience a pressure drop equal to (ΔP). The pressure at the outlet of the combustion chamber and the fuel-to-air ratio are given by the following:

$$P_{04} = (1 - \Delta P)\, P_{03}$$

$$f = \frac{Cp_{cc}T_{04} - Cp_c T_{03}}{\eta_b Q_R - Cp_{cc}T_{04}} \tag{6.5}$$

4. *Turbine:* It is not easy here to determine the outlet pressure and temperature of the turbine. The reason is that the turbine here drives both the compressor and propeller. The portion of each is not known in advance. Let us first examine the power transmission from the turbine to the propeller as illustrated in Figure 6.11.

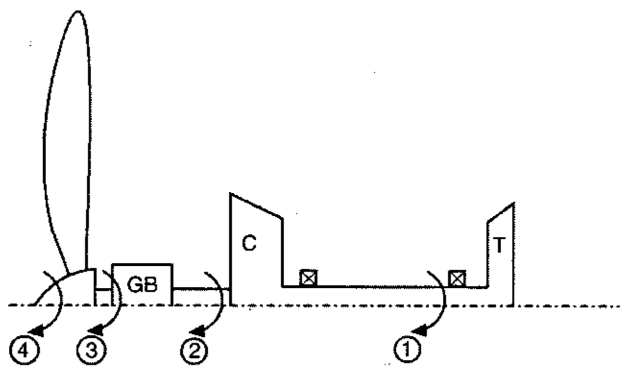
The output power from the turbine is slightly less than the extracted power owing to friction of the bearings supporting the turbine. This loss is accounted for by the mechanical efficiency of the turbine (η_{mt}). Moreover, the mechanical losses encountered in the bearings supporting the compressor are accounted for by the compressor mechanical efficiency (η_{mc}). The difference between both the turbine and compressor powers is the shaft power delivered to the reduction gear box where additional friction losses are encountered and accounted for by the gearbox mechanical efficiency (η_g). Finally the output power available from the propeller is controlled by the propeller efficiency (η_{pr}). Now, Figure 6.12 illustrates the enthalpy–entropy diagram for the expansion processes through both the turbine and the exhaust nozzle. It has been shown by Lancaster [1] that there is an optimum exhaust velocity that yields the maximum thrust for a given flight speed, turbine inlet temperature, and given efficiencies. Now let us define the following symbols as shown in Figure 6.12. Δh is the enthalpy drop available in an ideal (isentropic) turbine and exhaust nozzle and, $\alpha\Delta h = \Delta h_{\text{ts}}$, which is the fraction of Δh that would be available from an isentropic turbine having the actual pressure ratio

$$\Delta h_{\text{ns}} = (1-\alpha)\Delta h$$

which is also the fraction of Δh that may be available from an isentropic nozzle. η_t is the isentropic efficiency of turbine and, η_n is the isentropic efficiency of the exhaust nozzle.

Now to evaluate these values from the following thermodynamic relations:

$$\Delta h = Cp_{\text{t}} T_{04}\left[1-\left(\frac{P_{\text{a}}}{P_{04}}\right)^{(\gamma_{\text{h}}-1)/\gamma_{\text{h}}}\right] \tag{6.6}$$



Power transimission

at (1) $\eta_{mt} W_t$

at (2) $\left(\eta_{mt} W_t - \frac{W_c}{\eta_{mc}}\right)$

at (3) $\eta_{gb}\left(\eta_{mt} W_t - \frac{W_c}{\eta_{mc}}\right)$

at (4) $\eta_{pr}\eta_g\left(\eta_{mt} W_t - \frac{W_c}{\eta_{mc}}\right)$

FIGURE 6.11 Power transmission through a single-spool turboprop engine.

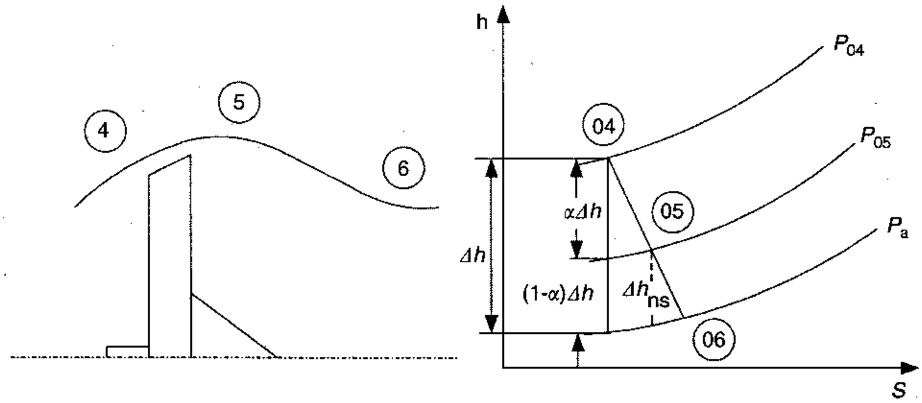


FIGURE 6.12 Expansion in the turbine and nozzle of a single-spool turboprop.

It was assumed in Equation 6.6 that the ratios between specific heats within the turbine and nozzle are constant, or

$$\gamma_t = \gamma_n = \gamma_h$$

The exhaust gas speed (U_e) is given by the following relation:

$$\frac{U_e^2}{2} = \eta_n(1-\alpha)\Delta h$$

$$U_e = \sqrt{2(1-\alpha)\Delta h \eta_n} \tag{6.7}$$

The procedure to be followed here is to deduce a mathematical expression for the thrust force generated by the propeller (T_{pr}) from the power generated by the propeller. Adding this thrust to the thrust generated by the exhaust gases, the total thrust is obtained as a function of (α). Differentiate with respect to (α) to obtain the optimum value of α giving the maximum thrust.

The propeller thrust T_{pr} is correlated to the propeller power by the following relation:

$$T_{pr} = \frac{\dot{m}_a \eta_{pr} \eta_g W_{shaft}}{U}$$

The shaft power is

$$W_{shaft} = \eta_{mt}(1+f-b)\Delta h_t - \frac{\Delta h_c}{\eta_{mc}}$$

where the turbine specific power, $\Delta h_t = \eta_t \alpha \Delta h$.

($\dot{m}_a$) is the air induction rate per second, and the fuel-to-air ratio and the bleed ratio are defined as: $f = (\dot{m}_f/\dot{m}_a)$ and $b = (\dot{m}_b/\dot{m}_a)$.

$$T_{pr} = \frac{\dot{m}_a \eta_{pr} \eta_g}{U}\left[(1+f-b)\eta_{mt}\eta_t \alpha \Delta h - \frac{\Delta h_c}{\eta_{mc}}\right] \tag{6.8}$$

The thrust force obtained from the exhaust gases leaving the nozzle is abbreviated as (T_n). If the fuel mass flow rate and the air bleed from the compressor are considered then it will be given by the following relation:

$$T_n = \dot{m}_a[(1+f-b)U_e - U]$$

Total thrust

$$T = T_{pr} + T_n$$

$$\frac{T}{\dot{m}} = \frac{\eta_{pr}\eta_g}{U}\left[(1+f-b)\eta_{mt}\eta_t\alpha\Delta h - \frac{\Delta h_c}{\eta_{mc}}\right] + \left[(1+f-b)\sqrt{2(1-\alpha)\eta_n\Delta h} - U\right] \quad (6.9)$$

Maximizing the thrust T for fixed component efficiencies, flight speed (U), compressor-specific power Δh_c, and expansion power Δh yields the following optimum value of (α_{opt}):

$$\alpha_{opt} = 1 - \frac{U^2}{2\Delta h}\left(\frac{\eta_n}{\eta_{pr}^2\eta_g^2\eta_{mt}^2\eta_t^2}\right) \quad (6.10)$$

Substituting this value of (α) in Equation 6.9 gives the maximum value of the thrust force. The corresponding value of the exhaust speed is given by the following equation:

$$U_e = U\frac{\eta_n}{\eta_{pr}\eta_g\eta_{mt}\eta_t} \quad (6.11)$$

6.3.2 Two-Spool Turboprop

A schematic diagram of a two-spool engine having a free power turbine together with its temperature–entropy diagram is shown in Figures 6.13 and 6.14. The low-pressure spool is composed of the propeller and the free power turbine while the high-pressure spool is composed of the compressor and the high-pressure or gas generator turbine.

The different components are examined here.

1. *Intake:* The same relations for the outlet pressure and temperature in the single spool; Equations 6.1 and 6.2 are applied here.

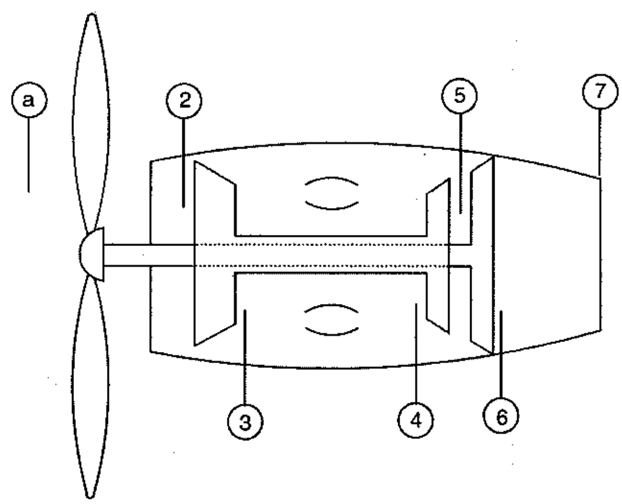


FIGURE 6.13 Layout of a free power turbine turboprop engine.

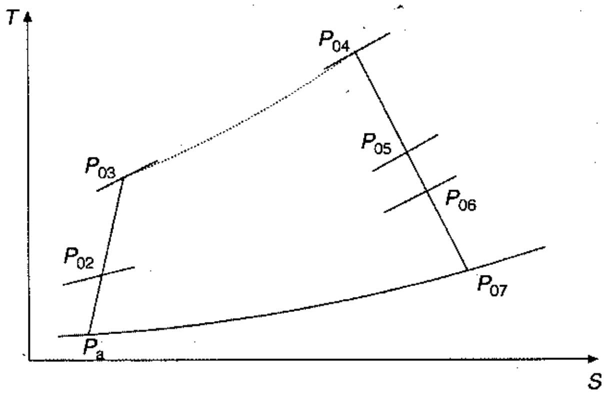


FIGURE 6.14 Temperature–entropy diagram for a free power turbine.

2. *Compressor:* The same relations in Equations 6.3 and 6.4 are applied here. The specific work of compressor (the work per kg of air inducted into the engine) is

$$\Delta h_c = Cp_c(T_{03} - T_{02})$$

3. *Combustion chamber:* The fuel-to-air ratio is obtained from the same relation, namely, Equation 6.5.

4. *Gas generator turbine:* An energy balance between the compressor and this high pressure turbine gives

$$\frac{\Delta h_c}{\eta_{mc}} = \eta_{mt}\Delta h_t \tag{6.12a}$$

The specific work generated in the turbine of the gas generator is

$$\Delta h_t = Cp_t(T_{04} - T_{05})(1 + f - b) \tag{6.12b}$$

From Equations 6.12a,b with known turbine inlet temperature, the outlet temperature (T_{05}) is calculated from the following relation:

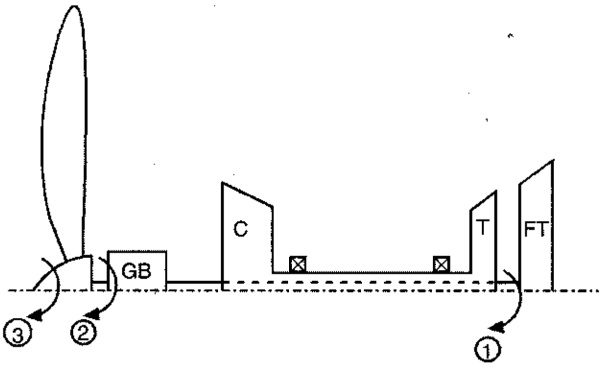
$$T_{05} = T_{04} - \frac{Cp_c(T_{03} - T_{02})}{Cp_t\eta_{mc}\eta_{mt}(1 + f - b)}$$

Moreover, from the isentropic efficiency of the gas generator turbine, the outlet pressure (P_{05}) is calculated from the relation given below:

$$P_{05} = P_{04}\left[1 - \left(\frac{T_{04} - T_{05}}{\eta_t T_{04}}\right)\right]^{\frac{\gamma_t}{\gamma_t - 1}}$$

5. *Free power turbine:* Figure 6.15 illustrates the power flow from the free turbine to the propeller. The work developed by the free power turbine per unit mass inducted into the engine is

$$\Delta h_{ft} = Cp_{ft}(1 + f - b)(T_{05} - T_{06})$$



at (1) $\eta_{mft} W_{ft}$

at (2) $\eta_{gb} \eta_{mft} W_{ft}$

at (3) $\eta_{pr} \eta_{gb} \eta_{mft} W_{ft}$

FIGURE 6.15 Power transmission through a double-spool turboprop engine.

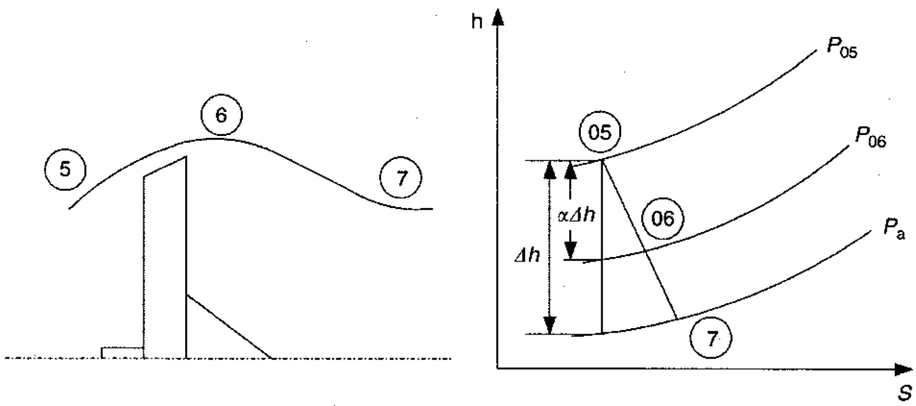


FIGURE 6.16 Expansion in the turbine and nozzle of a double-spool turboprop.

As noticed from the above equation and as previously explained in Section 6.3.1, the temperature (T_{06}) is unknown and cannot be calculated. Then a procedure similar to that described in Section 6.3.1 will be followed. Referring to Figure 6.16, which defines the successive expansion processes in the free power turbine and the nozzle, we have Δh = enthalpy drop available in an ideal (isentropic) turbine and exhaust nozzle; a full expansion to the ambient pressure is assumed in the nozzle ($P_7 = P_a$). Δh is then calculated as given below:

$$\Delta h = Cp_{\rm h} T_{05} \left[1 - \left(\frac{P_7}{P_{05}} \right)^{\frac{\gamma_{\rm h}-1}{\gamma_{\rm h}}} \right]$$

where $Cp_{\rm h} = Cp_{\rm t} = Cp_{\rm n}$ and $\gamma_{\rm h} = \gamma_{\rm t} = \gamma_n$.

$\alpha \Delta h = \Delta h_{\rm fts}$, which is the fraction of Δh that would be available from an isentropic free power turbine having the actual pressure ratio

$$\Delta h_{{\rm f}t} = \eta_{{\rm f}t} \Delta h_{{\rm f}t{\rm S}}$$

where η_{ft} is the isentropic efficiency of the free power turbine.

Following the same procedure described above to determine the optimum α, the propeller thrust and the exhaust thrust are determined from the following relations:

$$T_{\text{pr}} = \frac{\dot{m}_{\text{a}}\eta_{\text{pr}}\eta_{\text{g}}}{U}[(1+f-b)\eta_{\text{mft}}\eta_{\text{ft}}\alpha\Delta h] \tag{6.13}$$

$$T_{\text{n}} = \dot{m}_{\text{a}}\left[(1+f-b)U_{\text{e}} - U\right]$$

The total thrust is then given by,

$$T = T_{pr} + T_n$$

$$\frac{T}{\dot{m}} = \frac{\eta_{\text{pr}}\eta_{\text{g}}}{U}[(1+f-b)\eta_{\text{mft}}\eta_{\text{ft}}\alpha\Delta h] + [(1+f-b)\sqrt{2(1-\alpha)\eta_{\text{n}}\Delta h} - U] \tag{6.14}$$

where η_{mft} is the mechanical efficiency of the free power turbine.

Maximizing the thrust T for fixed component efficiencies, flight speed U and Δh yield the following optimum value of (α_{opt})

$$\alpha_{\text{opt}} = 1 - \frac{U^2}{2\Delta h}\left(\frac{\eta_{\text{n}}}{\eta_{\text{pr}}^2\eta_{\text{g}}^2\eta_{\text{mft}}^2\eta_{\text{ft}}^2}\right) \tag{6.15}$$

Substituting this value of (α) in Equation 6.14 gives the maximum value of the thrust force. The corresponding value of the exhaust speed is given by the following equation:

$$U_{\text{e}} = U\frac{\eta_{\text{n}}}{\eta_{\text{pr}}\eta_{\text{g}}\eta_{\text{mft}}\eta_{\text{ft}}} \tag{6.16}$$

The outlet conditions at the free turbine outlet are easily calculated from the known value of (Δh) and (α_{opt}).

6.4 ANALOGY WITH TURBOFAN ENGINES

Turboprop engines are analogous to high bypass turbofan engines. The propeller itself is an unducted fan with a bypass ratio equal to or greater than 25. Considering the puller type turboprop engines, the air flow through the propeller is slightly accelerated and thus acquires speed (u_1) slightly higher than the aircraft flight speed (u_0). The momentum difference between the inlet flow and outlet flow through the propeller produces the propeller thrust. Next this accelerated air passes through the engine core and accelerated to higher speeds (u_{e}). The momentum difference between the outlet and inlet core flow results in the core thrust. The thrust force is given by the relation:

$$T = \dot{m}_0[(u_1 - u_0)] + \dot{m}_{\text{a}}[(1+f-b)u_{\text{e}} - u_1] \tag{6.17a}$$

Introducing the bypass ratio (β) into Equation (6.17a) to get the following:

$$T = \dot{m}_{\text{a}}[\beta u_1 + (1+f-b)u_{\text{e}} - (1+\beta)u_0] \tag{6.17b}$$

The specific thrust related to the engine core mass flow rate is given by

$$\frac{T}{\dot{m}_{\text{a}}} = [\beta u_1 + (1+f-b)u_{\text{e}}] + ((1+\beta)u_0) \tag{6.17c}$$